CHARACTERIZATIONS OF CIRCLE PATTERNS AND CONVEX POLYHEDRA IN HYPERBOLIC 3-SPACE

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ABSTRACT. In this paper we consider the characterization problem of convex polyhedrons in the three dimensional hyperbolic space $\mathbb{H}^3$. Consequently we can give a characterization of circle patterns in the Riemann sphere with *dihedral angle* $0 \quad < \quad$. That is, for any circle pattern on $\hat{\mathbb{C}}$, its quasiconformal deformation space can be naturally identified with the product of the Teichm ̈

For any pair of adjacent boundary cone points $\{c_1, c_2\}$ with distance , we use

$[c_1; c_2]$

g/F2 4.=963 T

Theorem 0.7. *Assume that G*

Theorem 1.1. *A metric space* $(Q; g)$ *is the polar image of a finite volume polyhedron if and only if it satisfies the following conditions:*

2. CIRCLE PATTERNS

For any 0

Then Theorem 0.4 and Theorem 0.5 imply the following two results.

Theorem 2.1.

conditions 1 6 in Section 0. Obviously it satisfies the conditions 1; 2; 5 in Section

Lay down a regular hexagonal packing of circles in $\mathbb{C}$, say each of radius $1/n$.

By a small translation we can move the circle packing so that7(cie)-he o=

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In general, it is hard to determine the combinatorial structure of the polyhedron $\tilde{P}_n$

Proof. Fix A_3 as the origin. We also assume that A_2 lies on the positive real axis. Let z denote the complex coordinate of A_1. Hence z

angle . We will call these degenerate edges. Under the polar map : S^2_1 : S

Here $P(v_0)$ denote some boundary circle of I_1 such that $z_0 \in \partial I_1 \cap P(v_0)$. See [13, 5]. It is immediately that $(P'(v_{j_0}))/(P(v_{j_0})) < |{}'_1(z_0)| < 1$, which is a contradiction to (6). If z_0 is the tangent point of two adjacent circles $P(v_j); P(v_{j+1})$, a simple calculation shows that

$$|{}'_1(z_0)| = \left(\frac{1}{(P(v_j))} + \frac{1}{(P(v_{j+1}))}\right)\left(\frac{1}{(P'(v_j))} + \frac{1}{(P'(v_{j+1}))}\right)^{-1}. \tag{8}$$

cone pints c_v