ON THE MINIMAL FACTORIZATION OF THE HIGHER DIMENSIONAL QUASICONFORMAL MAPS

ZHENGXU HE & JINSONG LIU

ABSTRACT. With the aid of the logarithmic spiral mapping

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H. Grötzsch [8] first introduced plane quasiconformal homeomor-

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any given $0 < s < 1$, we have

$$f \quad \| have$$

2. Basic Materials

A quasiconformal homeomorphism $f : U \to V$ possesses the following properties, see e.g. [24].

(1). f is A. C. L (Absolutely Continuous on Lines). Also it is differentiable with Jacobian $J_f({}^{3}) > 0$ almost everywhere;

where $¸_1$ ¸ $¸_2$ ¸ ¢¢¢ ¸ $¸_n$ > 0. Denote

$$Q = A^T ¢ P^T ¢ \mathrm{diag}\,(¸_1^{i\,1}; ¸_2^{i\,1}; ¢¢¢ ; ¸_n^{i\,1}):$$

Then $Q^T ¢ Q = I_n$ (the n £ n identity matrix). Consequently/F2 10.909 Tf 10.06 6f02l49.1148 1.909

Consider the diagram

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We assume, by contradiction, that

(9) $$f_{3; ¸} j_{U_0} = f_2 \pm f_1;$$

for some K^s-quasiconformal map f_1 and $K^{1 i s}$-quasiconformal map f_2, where $0 < s < 1$.

Now we have the following result. Its proof will be postponed to Section 4.

Lemma 3.1. For almost all $^3 = (z; t)$ 2 U_0, there exist $P(^3)$ 2

Choose the closed curve $C_r \equiv \{|z| = r/2\} \times \{0\} \subset U_0$. Then the open curve $C_r' \equiv C_r \setminus \{z = -r/2\} \subset U'$

such that

$$\begin{aligned} Df_1(\xi) &= P_1(\xi)\cdot \operatorname{diag}\big(\lambda_1(\xi);\ \lambda_2(\xi);\ \lambda_3(\xi)\big)\cdot Q_1(\xi);\\ Df_2(\eta) &= P_2(\eta)\cdot \operatorname{diag}\big(\gamma_1(\eta);\ \gamma_2(\eta);\ \gamma_3(\eta)\big)\cdot Q_2(\eta);\end{aligned}$$

with $\lambda_1(\xi) \geq \lambda_2(\xi) \geq \lambda_3(\xi) > 0$ and $\gamma_1(\eta) \geq \gamma_2(\eta) \geq \gamma_3(\eta) > 0$. By (6) it follows that

$$\frac{\lambda_1(\xi)}{\lambda_3(\xi)} \leq K^s; \quad \frac{\gamma_1(\eta)}{\gamma_3(\eta)} \leq K^{1-s}; \quad a.e.$$

From $Df_{3;\theta}(\xi) = Df_2(f_1(\xi))\cdot Df_1(\xi)$, we deduce that

$$\begin{aligned}&\left(A(\theta;\mu')_{2\times 2}\right)\begin{pmatrix}\sqrt{K} & & \\ & 1/\sqrt{K} & \\ & & 1/\sqrt{K}\end{pmatrix}\left(B(\theta;\mu)_{2\times 2}\right)\\ = \;& P_2(\eta)\cdot\begin{pmatrix}\gamma_1(\eta) & & \\ & \gamma_2(\eta) & \\ & & \gamma_3(\eta)\end{pmatrix}\cdot Q_2(\eta)\cdot P_1(\xi)\cdot\begin{pmatrix}\lambda_1(\xi) & & \\ & \lambda_2(\xi) & \\ & & \lambda_3(\xi)\end{pmatrix}\cdot Q_1(\xi);\end{aligned}$$

where $\eta = f_1(\xi)$. That is, for a.e. $\xi \in U_0$,

$$\begin{aligned}&\begin{pmatrix}\sqrt{K} & & \\ & 1/\sqrt{K} & \\ & & 1/\sqrt{K}\end{pmatrix}\\ = \;& T_1(\xi)\cdot\begin{pmatrix}\gamma_1(\eta) & & \\ & \gamma_2(\eta) & \\ & & \gamma_3(\eta)\end{pmatrix}\cdot T_2(\xi)\cdot\begin{pmatrix}\lambda_1(\xi) & & \\ & \lambda_2(\xi) & \\ & & \lambda_3(\xi)\end{pmatrix}\cdot T_3(\xi);\end{aligned}$$

(16)

where

$$T_1(\xi) = \left(A(\theta;\mu')_{2\times 2}\right)^{-1}\cdot P_2(f_1(\xi)) \in SO(3);$$

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$T_2(\xi) = Q_2($

Similarly, by considering the actions on the column vector $(0; 0; 1)^T$, we have

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